Semianalytical Technique for Sensitivity Analysis of Unsteady Aerodynamic Computations

Durbha V. Murthy*
University of Toledo, Toledo, Ohio 43606
and
Krishna Rao V. Kaza†
NASA Lewis Research Center, Cleveland, Ohio 44135

A semianalytical approach is developed for the sensitivity analysis of linear unsteady aerodynamic loads. The semianalytical approach is easier to implement than the analytical approach. It is also computationally less expensive than the finite-difference approach when used with panel methods that require a large number of panels. The semianalytical approach is applied to an isolated airfoil in a two-dimensional flow and rotating propfan blades in three-dimensional flow. Sensitivity coefficients with respect to non-shape-dependent variables are shown for some cases. It is expected that the semianalytical approach will be useful in aeroelastic design procedures, particularly when mistuning is present, and also that it is potentially useful for shape sensitivity analysis of linear unsteady aerodynamics.

Nomenclature

A = domain of integration
AN = analytical approach
b_f = parameter denoting coupling of first and second propfan normal modes
C = matrix of aerodynamic influence coefficients
$f(x)$ = function to be differentiated with respect to x
FD = finite-difference approach
$H_1^{(2)}$ = first-order Hankel function of the second kind
$\boldsymbol{K}$ = generalized stiffness matrix
K = kernel function
k = reduced frequency
$\bar{L}$ = normal to blade helical surface
$\boldsymbol{M}$ = generalized mass matrix
M = freestream Mach number
$\boldsymbol{Q}$ = matrix of generalized aerodynamic forces
Q_{nm} = generalized force in nth mode due to motion in mth mode
$\boldsymbol{q}$ = generalized coordinate vector
S = propfan advance ratio (tip speed/axial speed)
SA = semianalytical approach
U = freestream fluid velocity
U_r = flow velocity relative to a point on the blade surface
v_n = complex amplitude of normal velocity fluctuation
W = normal velocity distribution on the lifting surface, $4\pi|\bar{L}|\, v_n\, e^{i\bar{\omega}\bar{\theta}}/U$ in the case of propfan aerodynamics
W_m = normal velocity distribution in mth mode
x = variable parameter
x_α = pitch-axis location
α = parameter
β = $\sqrt{1-M^2}$
Δ = prefix denoting finite-difference increment
Δp = pressure differential across the blade surface
$\overline{\Delta p}$ = $\dfrac{S^2}{\rho_0 U^2}\Delta p\, e^{i\bar{\omega}\bar{\theta}}$, called "pressure load parameter"
Δp_m = pressure differential across the blade surface due to motion in mth mode
δ_n = normal displacement in nth mode
ϵ = nondimensional control point location
$\bar{\theta}$, $\bar{\theta}_0$ = angular coordinates
ξ = dimensionless chordwise pressure station coordinate
ρ_0 = air density
τ = symbol for CPU times
φ = computed value of sensitivity coefficient of f with respect to x
Ω = propfan rotational speed
ω = blade vibration frequency
$\bar{\omega}$ = ω/Ω
$'$ = variation

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*NASA Resident Research Associate. Senior Member AIAA.
†Senior Research Engineer. Associate Fellow AIAA.

Introduction

THE computation of derivatives of response quantities with respect to design parameters, known as sensitivity analysis, plays an important role in developing reliable and efficient procedures for design optimization of practical aerospace structures. Integrating the structural and aerodynamic design processes for developing better aerospace structures in an automated manner is gaining interest.[1] Aerodynamic sensitivity analysis is necessary in order to make possible an efficient interdisciplinary approach to the optimization of aerospace systems. However, the interest in sensitivity analysis has been mainly confined to structural applications.[2,3] Dwyer, Peterson and Brewer,[4] and Dwyer and Peterson[5] applied sensitivity analysis to boundary-layer flow equations to compute the effect of various parameters, though not in the context of design optimization. A 1986 paper,[6] stressing the need for aerodynamic sensitivity analysis, contained only one reference on the subject and none dealing with unsteady aerodynamics. However, several papers on aerodynamic sensitivity analysis appeared recently (e.g., Refs. 7–10) in the context of multidisciplinary optimization. Some capabilities at analytical sensitivity analysis of aerodynamic computations have been reported in the literature (e.g., Refs. 8, 9, and references cited therein), but these have been limited to steady aerodynamics. Yates[8] presented a technique of analytical sensitivity analysis applicable to unsteady aerodynamic computations. However, no numerical results were reported. Also, the published literature contains no references to aerodynamic sensitivity analysis of rotary wing systems, such as propfans. The prediction of steady and unsteady aerodynamic loads on these systems is still a subject of intense research. In addition, the coupling of computational aerodynamic codes with structural analysis and optimization programs is not straightforward.

The generalized unsteady aerodynamic loads on a structure are, in general, functions of the flow conditions, the structural geometry, and the structural motion. The derivatives of the generalized aerodynamic forces, with respect to the variables representing the structural geometry and motion, are useful in aeroelastic structural design procedures. Their computation constitutes the aerodynamic shape sensitivity analysis. Shape sensitivities are immensely useful in automated design of aircraft wings and rotating blades. For example, shape sensitivities can be used to compute optimal sweep distribution on propfan blades.

These sensitivity derivatives can be calculated using a straightforward finite-difference approach. However, experience in other engineering disciplines suggests that a semianalytical approach may be more computationally efficient and easier to implement with complex aerodynamic codes. The objective of the present paper is to present a semianalytical approach for sensitivity analysis of subsonic unsteady aerodynamics used in flutter analysis, for the purpose of computing the derivatives of the generalized unsteady aerodynamic forces.

As a first step, only non-shape-dependent variables, representing the flow conditions and the structural motion, are considered. Such variables include freestream Mach number, vibration frequency, mode shape, and rotational speed. In addition to aeroelastic optimization, the derivatives with respect to flow conditions and structural motion can also be useful in computational schemes for aeroelastic analysis[11-13] and in computing the derivatives of the flutter Mach number and flutter frequency with respect to structural design variables.[14-16] In the following, the problem of numerical sensitivity analysis is introduced, and the three possible approaches to the problem (finite-difference, analytical, and semianalytical) are described. The semianalytical approach is then described in further detail in the context of unsteady aerodynamic computations using panel methods. Finally, the application of sensitivity analysis to isolated airfoils in two-dimensional flow and to single rotation propfan blades in three-dimensional flow is presented along with some typical results.

Aerodynamic Sensitivities in Aeroelastic Analysis and Optimization

The linearized homogenous equations of motion in frequency domain can be written in the general form[17]

$$[-\omega^2 M + K - Q]q = 0 \tag{1}$$

In two-dimensional applications, the nondimensional parameter k is generally used in place of ω.

The matrices M and K are dependent on the structural properties and blade shape. The matrix Q depends, in addition, on the frequency ω and on flow parameters such as the Mach number M. Most procedures of finding the aeroelastic stability boundaries involve the solution of either the nonlinear eigenvalue problem, e.g.,[18]

$$B(\omega, M)q = 0 \tag{2}$$

where $B = [-\omega^2 M + K - Q]$ or a series of linear eigenvalue problems given by, e.g.,[19]

$$[K - Q_a - \omega^2 M]q = 0 \tag{3}$$

where the matrix Q_a represents Q evaluated at an assumed frequency and Mach number. The eigenvalues of both Eq. (2) and Eq. (3) are functions of the generalized unsteady aerodynamic forces in Q.

Equation (2) is generally solved by a Newton-like iteration scheme,[13] which requires the aerodynamic sensitivities $\partial Q/\partial\omega$ and $\partial Q/\partial M$. In the case of aeroelastic optimization, constraints are typically placed on the eigenvalues of either Eq. (2) or Eq. (3). Thus, the calculation of constraint derivatives, essential for any numerical optimization procedure, requires the evaluation of the aerodynamic sensitivities $\partial Q/\partial\alpha$, where α is one of several design parameters.

Numerical Sensitivity Analysis

In the finite-difference approach, the sensitivity coefficient of a function f of a parameter x is evaluated by repeating the analysis after incrementing each parameter. That is,

$$\varphi_{\mathrm{FD}}(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{4}$$

where Δx is the finite-difference step size. Although only first-order forward differences are considered, the comments apply equally to others. The finite-difference approach is computationally expensive, and accuracy is very much dependent on the selected step size Δx and roundoff error accumulation. The proper selection of a finite-difference step size is not trivial. However, the finite-difference approach is easy to implement.

The analytical approach to the problem of sensitivity analysis consists of evaluating the sensitivity coefficient by direct analytical differentiation. That is,

$$\varphi_{\mathrm{AN}}(x) = \frac{\mathrm{d}f}{\mathrm{d}x} \tag{5}$$

The analytical approach obviously results in the exact sensitivity coefficient. However, it is difficult to implement because the function $f(x)$ tends to be very complicated in practice and, in general, cannot be directly differentiated. Furthermore, many response functions of interest are computed by special purpose programs and analytical differentiation is impractical. The tedium involved in direct differentiation of a compli-

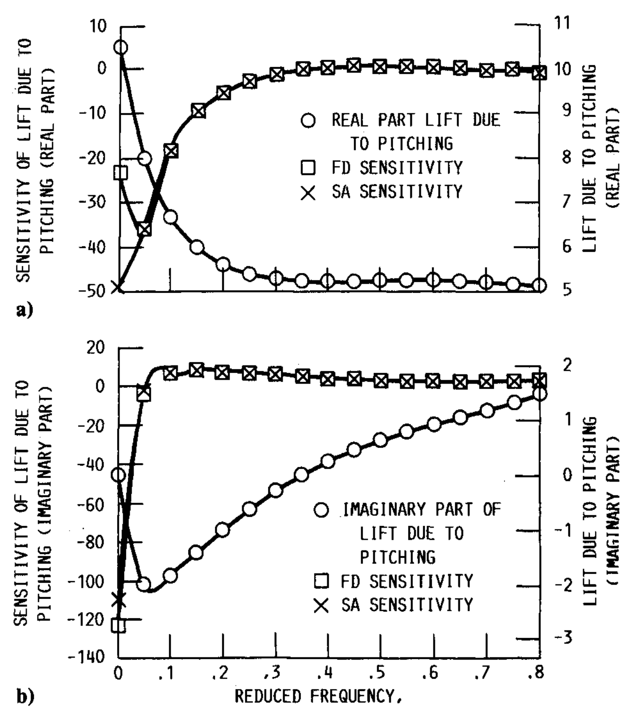

Fig. 1 Sensitivity of generalized forces to reduced frequency.

cated function can be somewhat alleviated by methods such as the one described in Ref. 20. This method still involves substantial modification of the original program and could be costly to implement. The precompilers described in Ref. 21 promise to reduce the cost of implementation of the analytical approach. However, these precompilers are yet to be tested in structural or aerodynamic applications. In addition, in unsteady aerodynamic applications using panel methods, the cost of computing $\varphi_{AN}(x)$ is not much smaller than the cost of computing $f(x)$. Thus, the cost of their implementation is not offset by a substantial reduction in computational effort, as is often the case in structural applications. Hence, analytical methods of sensitivity analysis are not easily justified in unsteady aerodynamic applications.

The semianalytical approach consists of analytical differentiation of the original function with respect to an intermediate function, the derivative of which is then evaluated by numerical differentiation. Thus, if $f(x) = f[g(x)]$, then

$$\varphi_{SA}(x) = \frac{df}{dg} \cdot \frac{g(x + \Delta x) - g(x)}{\Delta x} \tag{6}$$

The semianalytical approach combines the efficiency of the analytical approach with the ease of implementation of the finite-difference approach. The most beneficial combination results if most of the computational effort of evaluating the function $f(x)$ is contained in computing $f(g)$ and most of the analytical complexity is contained in computing $g(x)$. It can also be sometimes expected to result in more accurate derivatives than the finite-difference approach. For example, if $g(x)$ is nearly linear, then the SA approach results in much more accurate derivatives than the FD approach.

Semianalytical Approach to the Sensitivity Analysis of Unsteady Aerodynamics

The integral equation expressing the relation between the upwash and the pressure distribution can be written as[17]

$$W = \int_A \Delta p \, K \, dA \tag{7}$$

Note that K may be formulated for steady, unsteady, two-dimensional, three-dimensional, incompressible, or compressible flow. Thus, the techniques are applicable to any of the preceding flow conditions, though the implementation presented herein is limited to two selected aerodynamic models. We consider unsteady compressible flow where structural motion is simple harmonic. The discussion, however, is also applicable to nonharmonic motion by replacing the real variable ω, representing frequency, by the complex variable s, representing the Laplace transform variable. The normal velocity W depends on the assumed motion of the lifting surface (structural motion) and is considered known. The kernal function for nonrotating systems depends on the flow Mach number and frequency of motion. For rotating systems, it also depends on the rotational speed. Given the coordinate position of the elemental area dA, the kernel function can be computed at any point on the lifting surface. Knowing W and K, the pressure distribution Δp is obtained by inverting the preceding integral equation. For aeroelastic applications, the primary interest lies not in the pressure distribution but in the generalized force contained in the matrix Q, given by

$$Q_{nm} = \int_A \Delta p_m \delta_n \, dA \tag{8}$$

If steady aerodynamic displacements are neglected, the normal displacement δ_n is independent of the assumed vibration frequency ω and the flow conditions.

We seek the derivatives of the generalized aerodynamic forces, contained in the matrix Q, with respect to parameters representing flow conditions and structural vibratory motion but independent of the domain of integration A. Because the variation of generalized forces with respect to the mode shape is simpler, mode shape sensitivity is considered separately. First, consider parameters that do not depend on the mode shape. Let α be one such parameter. Differentiating Eq. (8) with respect to α,

$$\frac{\partial Q_{nm}}{\partial \alpha} = \int_A \frac{\partial \Delta p_m}{\partial \alpha} \delta_n \, dA \tag{9}$$

Thus, the sensitivity of the generalized force is equal to the generalized force acting on the blade resulting from a pressure differential across the blade surface given by $\partial \Delta p_m / \partial \alpha$. We refer to $\partial \Delta p_m / \partial \alpha$ as the "pseudopressure differential" corresponding to the parameter α.

To obtain the pseudopressure differential, we differentiate Eq. (7) to get

$$\frac{\partial W}{\partial \alpha} - \int_A \Delta p \frac{\partial K}{\partial \alpha} \, dA = \int_A \frac{\partial \Delta p}{\partial \alpha} K \, dA \tag{10}$$

Equation (10) is an integral equation in the unknown $\partial \Delta p / \partial \alpha$ and is identical to Eq. (7) if the left side and $\partial \Delta p / \partial \alpha$ in Eq. (10) are replaced by W and Δp, respectively. Thus, the sensitivity of the pressure distribution is equal to the pressure distribution that gives rise to the normal velocity distribution, given by

$$\frac{\partial W}{\partial \alpha} - \int_A \Delta p \frac{\partial K}{\partial \alpha} \, dA$$

We refer to this velocity distribution as the "pseudo-upwash" distribution, corresponding to the parameter α. The pseudoupwash is analogous to the concept of the pseudoload used in the sensitivity analysis of static structural response.[3]

To evaluate the pseudoupwash, $\partial W / \partial \alpha$ and $\partial K / \partial \alpha$ must be computed. Whereas it is often less difficult to evaluate $\partial W / \partial \alpha$ analytically, the analytical evaluation of $\partial K / \partial \alpha$ is generally time-consuming and difficult to implement. Therefore, the pseudoupwash is evaluated by replacing the quantities $\partial W / \partial \alpha$ and $\partial K / \partial \alpha$ by their finite-difference approximations, $\Delta W / \Delta \alpha$ and $\Delta K / \Delta \alpha$, respectively. Considering forward differences,

$$\frac{\partial W}{\partial \alpha} \simeq \frac{\Delta W}{\Delta \alpha} = \frac{W(\alpha + \Delta \alpha) - W(\alpha)}{\Delta \alpha} \tag{11}$$

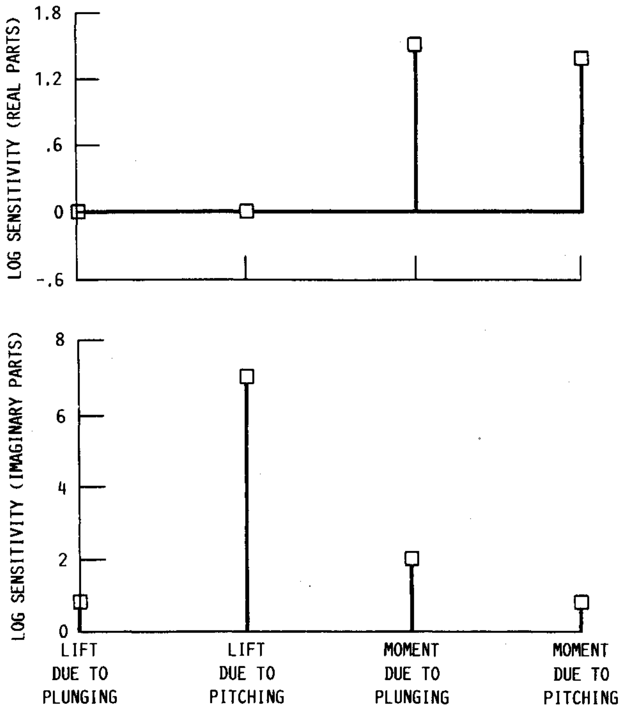

Fig. 2 Logarithmic sensitivity of generalized forces to reduced frequency.

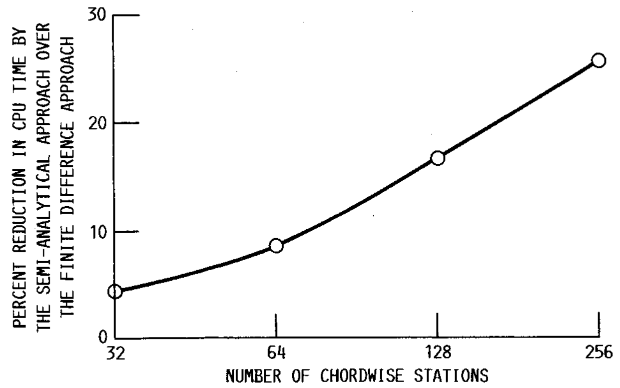

Fig. 3 Efficiency of the semianalytical approach over the finite-difference approach.

$$\frac{\partial K}{\partial \alpha} \simeq \frac{\Delta K}{\Delta \alpha} = \frac{K(\alpha + \Delta\alpha) - K(\alpha)}{\Delta \alpha} \tag{12}$$

For sensitivity analysis with respect to mode shapes, we first note that the pressure distribution is a linear functional and the generalized force a quadratic functional of the mode shape. Hence, we consider variations instead of derivatives. Denoting variations by primes, we obtain from Eq. (8)

$$Q'_{nm} = \int_A \Delta'_m \delta_n \, \mathrm{d}A + \int_A \Delta p_m \delta'_n \, \mathrm{d}A \tag{13}$$

and from Eq. (7)

$$W' = \int_A \Delta p'_m \, K \, \mathrm{d}A \tag{14}$$

so that, once again, the variation $\Delta p'_m$ of the pressure distribution can be obtained by using W'_m as the pseudoupwash distribution. Thus, for mode shape sensitivity analysis, we use Eqs. (13) and (14) instead of Eqs. (9) and (10), respectively.

For simple harmonic motion of an airfoil, the upwash can be approximated as[17]

$$W_m(P) = U_r \frac{\mathrm{d}\delta_m}{\mathrm{d}s}(P) + i\omega\delta_m(P) \tag{15}$$

so that

$$W'_m(P) = U_r \frac{\mathrm{d}\delta'_m}{\mathrm{d}s}(P) + i\omega\delta'_m(P) \tag{16}$$

where U_r is the flow velocity relative to the point P on the airfoil and s is the arc length along the blade surface at fixed span.

A popular way to solve the integral equation given by Eq. (7) for the pressure distribution is the family of panel methods. In these methods, the singular pressure distribution is approximated by a panelwise constant pressure distribution. The Kutta condition is implicitly satisfied by choosing specific control points. Panel methods can easily handle any planform and are most suitable for lifting surfaces of complex geometry. Fine paneling of the planform is necessary because of the rough approximation of panelwise constant pressure distribution.

Advantages of the Semianalytical Approach

Once a discretization scheme (such as a panel method) is devised, practically all of the analytical complexity in computing the generalized forces Q_{nm} is contained in the computation of the kernel function K. Thus, as discussed in the last section, a semianalytical sensitivity analysis scheme, using numerical differentiation for the kernel function derivative, is of considerable advantage in overcoming the implementation problems associated with the analytical approach. Also, the aerodynamic code used for analysis can be used for sensitivity analysis as well by simply replacing the real upwash distribution with the pseudoupwash distribution and the real pressure differential with the pseudopressure differential. This results in considerable simplicity in the implementation of the sensitivity analysis. The approach adopted here is similar to the semianalytical approach popular in the sensitivity analysis of static structural response[22,23] and implemented in some general purpose finite element programs.[24]

The efficiency of the semianalytical approach in comparison to the finite-difference approach depends on the size of the problem. We assume in the following that the integral operator is inverted by a direct method, such as Gauss elimination, rather than by an iterative method. For both the FD and SA approaches, iterative methods are more difficult to handle because of the presence of large truncation errors and errors arising from the early termination of iterative process and the different initial guesses used for the nominal and the perturbed solutions.[25]

The overall costs of computing the kernel function and of inverting the integral operator are roughly proportional to the square and the cube of the number of panels, respectively. Hence, if the number of panels is n, then the CPU time for the computation of the generalized forces can be expressed as

$$\tau = \tau_1 n^3 + \tau_2 n^2 \tag{17}$$

The first term represents the time required for inverting the integral operator, and the second term the time required for kernel function evaluations. The semianalytical approach utilizes the same inverted operator (available from the solution of the corresponding analysis problem) for all derivatives, in contrast to the finite-difference approach. Hence, the CPU times for the FD and the SA approaches of sensitivity analysis can be expressed as

$$\tau_{\mathrm{FD}} = \tau_1 n^3 + \tau_2 n^2 \tag{18}$$

$$\tau_{\mathrm{SA}} = \tau_2 n^2 \tag{19}$$

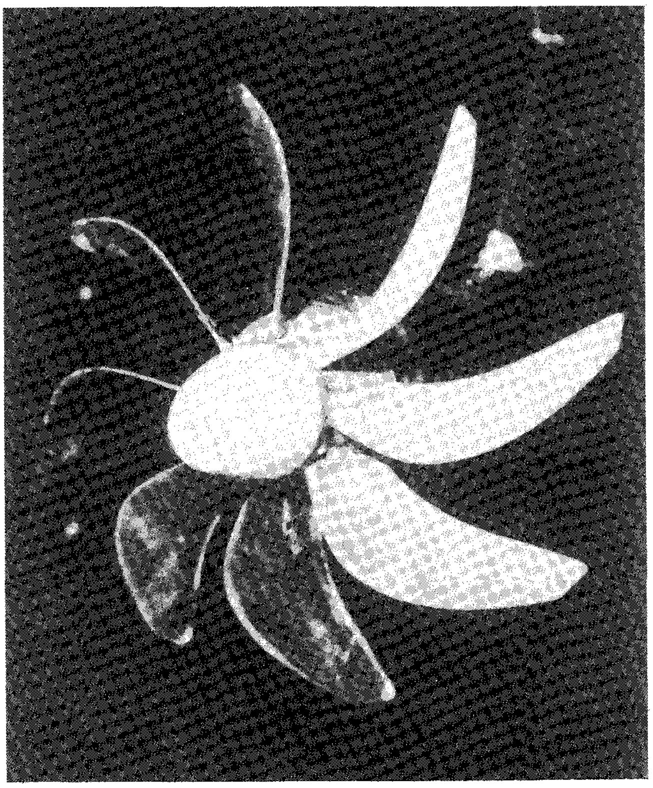

Fig. 4 The SR3C-X2 propfan model in the Lewis 8×6 ft wind tunnel.

In pressure formulations, for a moderate number of panels, the second term of Eq. (18), representing the kernel function computation, dominates the overall computational time, so that τ_{FD} and τ_{SA} are of the same order. However, as n becomes larger, the cost of the FD approach escalates more rapidly than that of the SA approach. Hence, when the number of panels is large, the efficiency gain of the SA approach over the FD approach can be substantial. A large number of panels is needed in the case of a complex geometry such as that of a propfan, especially if mistuning is present.[26] A further discussion of the computational cost of the solution of the integral equation is given by Clark and James.[27]

Applications

The semianalytical approach is applied to two unsteady aerodynamic models: 1) isolated airfoil in two-dimensional flow and 2) rotating propfan blades in three-dimensional flow. Sensitivity coefficients of generalized forces with respect to various parameters are calculated by both the FD and SA approaches and compared. For the purposes of computing the finite-difference derivatives, the step sizes were computed by a procedure described by Gill.[28] This procedure seeks to minimize the total error (the sum of roundoff and truncation errors) in the sensitivity coefficient using the roundoff error expected in the evaluation of function $f(x)$. The roundoff error in the evaluation of $f(x)$ is estimated by the difference in the values of $f(x)$ calculated in double precision and extended precision. All other calculations were carried out in double precision.

In the following, the logarithmic sensitivity function is defined as $x\,\varphi(x)/f(x)$. Thus, the sensitivity coefficient gives the absolute rate of change of the function $f(x)$, whereas the logarithmic sensitivity function gives a measure of the relative rate of change of $f(x)$.

Isolated Airfoil in Two-Dimensional Compressible Flow

The kernel function for this unsteady aerodynamic model is the Possio kernel,[29] given by

$$K(x,\xi) = -\frac{iMk}{8\beta}\, e^{-ik(x-\xi)} \int_{-\infty}^{k(x-\xi)/\beta^2} e^{i\lambda} H_1^{(2)}(M|\lambda|)\, \frac{d\lambda}{|\lambda|} \qquad (20)$$

where b is the blade semichord and x and ξ represent the dimensionless chordwise control and pressure stations, respectively. The Possio kernel is valid for subsonic Mach numbers. Bland[29] gives a full description of the computational aspects of the Possio kernal evaluation.

For the isolated airfoil, the parameters considered are the freestream Mach number M, the reduced frequency k, and the location of the pitch axis x_α. The nominal point chosen for this configuration corresponds to $M=0.8$, $k=0.10$, and $x_\alpha=-0.5$ (i.e., the pitch axis is at the quarter-chord). The downwash and pressure are computed at 32 stations along the chord. The finite-difference step sizes were selected by minimizing the estimated total error in the sensitivity coefficients of selected normal velocities, pressure differentials, and generalized forces. The estimated error in the derivatives[28] was less than 1.5% in all cases.

Figure 1 shows the variation of the generalized forces due to pitching motion and their sensitivity coefficients with respect to the parameter k. Both the real parts (in phase with the motion) and the imaginary parts (in quadrature with the motion) are shown in Figs. 1a and 1b, respectively. The axes on the right correspond to the function and those on the left to its sensitivity. The squares give the finite-difference (FD) sensitivities and the crosses the semianalytical (SA) sensitivities. Except at zero reduced frequency, the FD and the SA coefficients were in very good agreement. Also, the variations of the generalized forces are consistent with the calculated values of the respective sensitivity coefficients. At zero reduced frequency, a large difference between the FD and SA sensitivity coefficients of the real part of the generalized force was obtained. An interpolation of the generalized forces (as shown by the line joining the open circles in Figure 1a) indicates that the SA coefficient is substantially more accurate than the FD coefficient. Similar results were obtained for generalized forces due to plunging and for sensitivities with respect to M and x_α.

Figure 2 shows the logarithmic sensitivities, defined in the previous section, of the generalized forces with respect to k. At this nominal point, the imaginary part of the lift due to pitching is seen to be the most sensitive to reduced frequency, and the real parts of the lift due to pitching, as well as plunging, the least sensitive.

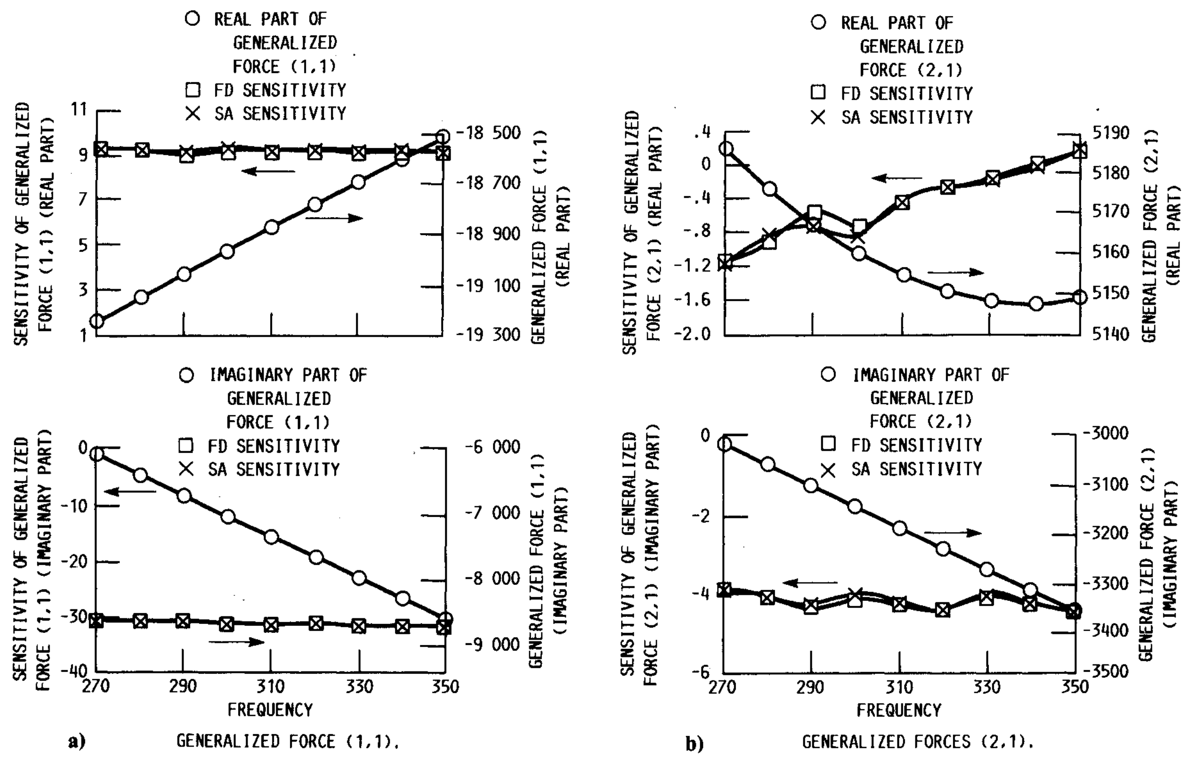

Fig. 5 Sensitivity of generalized forces to frequency.

To compare the efficiency of the semianalytical method with the finite-difference method, the percentage saving of CPU time achieved by the semianalytical approach is shown in Fig. 3 as a function of the number of chordwise stations. These CPU times were obtained in computing the sensitivity of generalized forces with respect to Mach number. The percentage saving in CPU time using the semianalytical approach ranges from 7.3%, when the number of chordwise stations is 32, to 25.4%, when the number is 256. The CPU times were obtained on the Amdahl 5860 computer using VS Fortran Version 2 compiler with no compiler optimization.

Propfan Blades in Three-Dimensional Compressible Flow

The second unsteady aerodynamic model studied is based on a three-dimensional linear subsonic lifting surface theory in frequency domain and was developed by Williams and Hwang[30] for aeroelastic analysis of propfans.[19]

The aerodynamic sensitivity analysis described below was implemented in a computer program called ASTROP3 at NASA Lewis Research Center. ASTROP3 is an aeroelastic analysis program developed for aeroelastic analysis of single rotation propfans.[19] An automated flutter search procedure,[13] requiring the generalized force sensitivities, is also incorporated in ASTROP3.

The integral equation expressing the relationship between the upwash and pressure distributions on a thin propfan blade is given by

$$W(P) = -\int_A \overline{\Delta p}(P_0)\frac{\partial}{\partial \Theta}[K(P,P_0)]\,\mathrm{d}A_0 \tag{21}$$

Note that the definition of the kernel function does not exactly correspond to that of the previous section.

Williams and Hwang[30] discretized the preceding integral equation by splitting the blade into n rectangular panels within each of which $\overline{\Delta p}$ is assumed constant. One control point is assigned to each panel at the midspan and 100ϵ percent chord position. The best value for ϵ was empirically found[30] to be 0.85. The discretization results in the algebraic system of equations given by

$$W = C\overline{\Delta p}$$

or

$$\overline{\Delta p} = C^{-1}W \tag{22}$$

where W is a vector of the values of W at chosen control points on each of the panels, $\overline{\Delta p}$ is a vector of the values of $\overline{\Delta p}$ on each of the panels, and C is a matrix of aerodynamic influence coefficients given by

$$c_{ij} = -\int_{A_j}\frac{\partial}{\partial \Theta}[K(P_i,P_0)]\,\mathrm{d}A_0 \tag{23}$$

where the subscripts i and j refer to the panel numbers.

The computation of the pseudopressure differential requires the derivative of the vector of the pressure load parameters, $\partial\Delta p/\partial\alpha$, which is obtained by differentiating Eq. (22):

$$\frac{\partial W}{\partial \alpha} = C\frac{\partial \overline{\Delta p}}{\partial \alpha} + \frac{\partial C}{\partial \alpha}\overline{\Delta p}$$

or

$$\frac{\partial \overline{\Delta p}}{\partial \alpha} = C^{-1}\left[\frac{\partial W}{\partial \alpha} - \frac{\partial C}{\partial \alpha}\overline{\Delta p}\right] \tag{24}$$

For mode-shape-dependent parameters,

$$\frac{\partial \overline{\Delta p}}{\partial \alpha} = C^{-1}\frac{\partial W}{\partial \alpha} \tag{25}$$

because the aerodynamic influence coefficients are independent of mode shapes.

For propfan blades, the parameters considered are the freestream Mach number M, the blade vibration frequency ω,

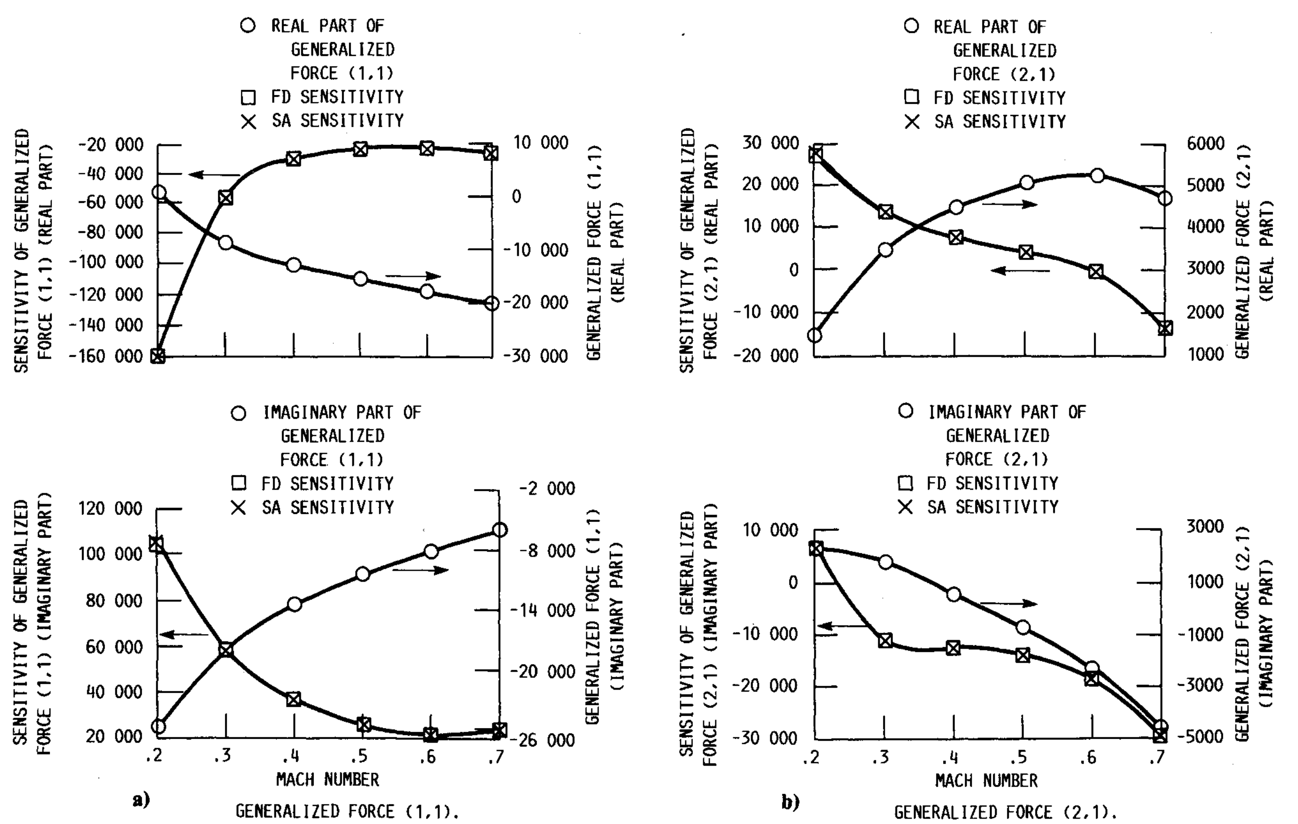

Fig. 6 Sensitivity of generalized forces to Mach number.

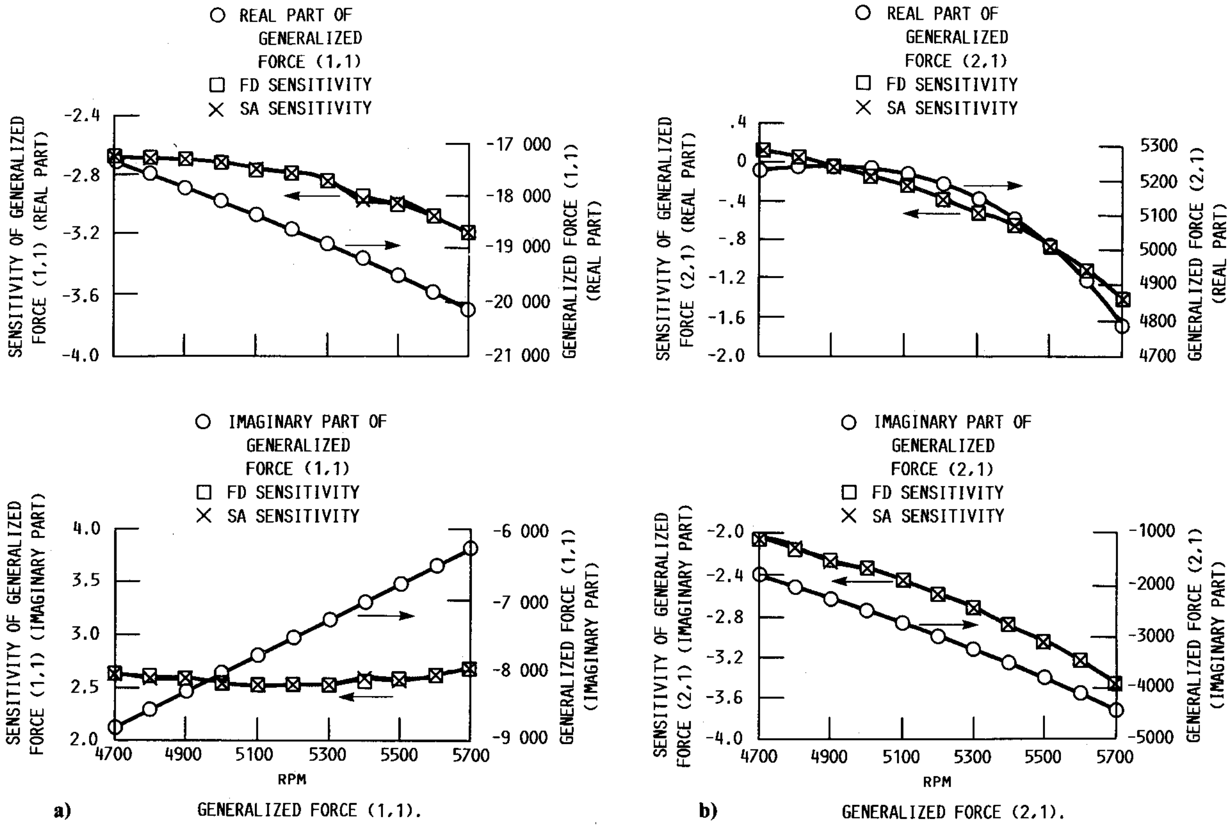

Fig. 7 Sensitivity of generalized forces to rotational speed.

the rotational speed Ω, the location of the control point ϵ, and a parameter b_f denoting the coupling of the first and second normal modes of the blade. The parameter b_f defines the normal displacement of the blade mode shape as $\delta(P) = b_f\delta_1(P) + (1 - b_f)\,\delta_2(P)$, where δ_1 and δ_2 are the normal displacements due to motion in the first and the second blade normal modes, respectively. Thus, the parameter b_f represents the coupling of the first two normal modes. In this formulation, the domain of integration A is dependent on the advance ratio. In order not to change the shape of the helical surface and, thus, maintain the non-shape-dependent nature of the variables, the advance ratio was artificially held constant while computing the derivatives with respect to the Mach number and the rotational speed. This limitation will be removed when the sensitivity analysis is extended to shape-dependent variables. The control point location ϵ is also considered a parameter because its value is chosen empirically and the sensitivity of the generalized forces to ϵ is of interest.

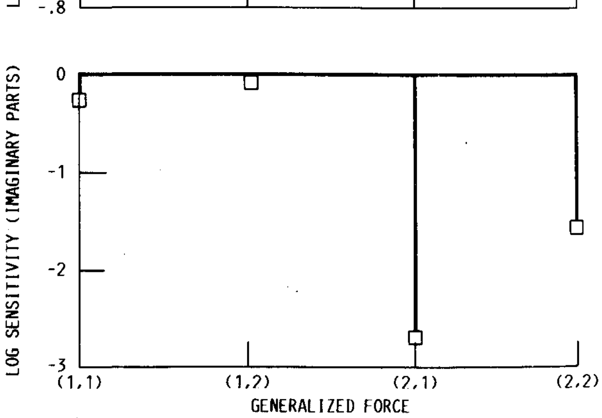

Fig. 8 Sensitivity of generalized forces to control point location.

The planform and the normal modes of the SR3C-X2 propfan blade,[19] shown in Fig. 4, were used as an example. The normal modes analysis was performed using COSMIC NASTRAN considering centrifugal deformations but not steady aerodynamic deformations. The planform was divided into 72 panels. The finite-difference step sizes were selected by minimizing the estimated total error in the sensitivity coefficients of selected normal velocities, influence coefficients, pressure differentials, and generalized forces. The estimated error in the derivatives[28] was less than 2.4% in all cases.

Sensitivity to Frequency, Mach Number, and Rotational Speed

The computed sensitivities with respect to the vibration frequency of the generalized forces due to motion in the first mode are shown in Fig. 5, along with the generalized forces on the right vertical axis. Figures 6 and 7 similarly show the sensitivities with respect to Mach number and the rotational speed, respectively. Once again, the FD and the SA sensitivities are in reasonable agreement. Both the FD and the SA sensitivities are sometimes affected by roundoff error, as evidenced by their wavy nature. This indicates that the step-size selection algorithm used[28] may be inadequate in some cases and an improved algorithm[31] should perhaps be employed.

Sensitivity to Empirical Parameters

An important application of sensitivity analysis is in judging the sensitivity of generalized forces to empirical parameters. Such sensitivity gives a measure of the confidence in the values selected for the empirical parameters. As an example of such an application, the sensitivities of the generalized forces are calculated with respect to the location ϵ of the control point on the panel at which the normal velocity due to structural motion is specified. Figure 8 shows the logarithmic sensitivity of generalized forces with respect to the control point location. It is observed that the generalized force could be expected to change by up to 3%, for a 1% change in the control point location. This is considered satisfactory. Finally, for the 72 panels used here, the percent reduction in CPU time from the semianalytical approach over the finite-difference approach was 8–13%. Greater reductions are expected for larger numbers of panels.

Concluding Remarks

A semianalytical approach to sensitivity analysis of linear unsteady aerodynamics is presented. Applications to isolated airfoils and propfan blades have shown that the semianalytical approach can be implemented relatively easily and that the semianalytical approach does not suffer from worse accuracy problems than the finite-difference approach. Preliminary studies show that the semianalytical approach will result in substantial savings in computational time for sensitivity analysis when the number of panels in propfan aeroelastic analysis is large. This is typically the case when mistuning is considered.

It is expected that the semianalytical approach will be useful for computing shape sensitivity derivatives of generalized unsteady aerodynamic forces. These derivatives are expected to be used in developing efficient analytical and design optimization procedures for complex aeroelastic structures.

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